**R09**

**Code No: D2104**

**JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD**
**M.TECH II - SEMESTER EXAMINATIONS, APRIL/MAY 2012**
**COMPUTATIONAL FLUID DYNAMICS**
**(THERMAL ENGINEERING)**

**Time: 3hours** **Max. Marks: 60**

**Answer any five questions**
**All questions carry equal marks**
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1. (a) Obtain a 5-point centre-difference scheme for $\frac{\partial^2 \phi}{\partial x^2}$ at grid-point *i* using $\phi_{i-2}, \phi_{i-1}, \phi_i, \phi_{i+1}, \phi_{i+2}$ and find its truncation error.
   (b) Perform the truncation error analysis for the standard Crank-Nicolson scheme for the 1-D heat conduction equation, and find its order of accuracy.

2. Do von-Neumann stability analysis on the CTCS (center time center space) scheme for the 1-D heat conduction equation and find its stability condition.
$$\frac{T_i^{n+1} - T_i^{n-1}}{2\Delta t} = \alpha \frac{T_{i+1}^n - 2T_i^n + T_{i-1}^n}{\Delta x^2}$$
Assume that the amplification factor, ‘a’ is the same for two sequential steps: i.e.
$$\frac{T_i^{n+1}}{T_i^n} = \frac{T_i^n}{T_i^{n-1}} = a .$$

3. (a) What are the different types of partial differential equations? Classify and describe their solution methods.
   (b) Find the types of the following partial differential equations.
      i. $\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$
      ii. $\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$
      iii. $\frac{\partial^2 T}{\partial t^2} = c^2 \frac{\partial^2 T}{\partial x^2}$
      iv. $\frac{\partial T}{\partial t} + c \frac{\partial T}{\partial x} = 0$

4. Describe the steps associated with (a) the pressure correction method and (b) simple algorithm for incompressible viscous fluid flow.

5. The equations of motion for free convection near a hot plate for incompressible flow are

$$\frac{du}{dx}+\frac{dv}{dy}=0$$

$$\frac{du}{dt}+u\frac{du}{dx}+v\frac{du}{dy}=-\frac{dp}{dx}+\nu\left(\frac{d^2u}{dx^2}+\frac{d^2u}{dy^2}\right)$$

$$\frac{dv}{dt}+u\frac{dv}{dx}+v\frac{dv}{dy}=-\frac{dp}{dy}-g\beta(T-T_l)+\nu\left(\frac{d^2v}{dx^2}+\frac{d^2v}{dy^2}\right)$$

$$\rho c_p\left(\frac{dT}{dt}+u\frac{dT}{dx}+v\frac{dT}{dy}\right)=k\left(\frac{d^2T}{dx^2}+\frac{d^2T}{dy^2}\right)$$

Introduce the dimensionless variables

$$u^*=\frac{u}{U},v^*=\frac{v}{U},p^*=\frac{p}{\rho U^2},t^*=\frac{tU}{L},x^*=\frac{x}{L},y^*=\frac{y}{L},T^*=\frac{T-T_l}{T_0-T_l},$$

where $L$ is the length of the plate; $U$ and $T_0$ are reference velocity and temperature respectively. $T_l$ is the temperature of the plate. Use these variables to non dimensionalise the free convection equations and define any dimensionless parameters which arise.

6. (a) Derive the second order accurate forward and backward upwinding schemes for $\frac{\partial \phi}{\partial x}$ .

   Assume that the distance between two grid points is h.

   (b) Derive the expression for vorticity at the wall in terms of stream function. The expression should contain the interior points only. One could use no-slip velocity boundary condition at the wall in deriving the expression.

7. Derive the Quasi one-dimensional compressible flow equations for flow through a nozzle. Explain the method of capturing the shock in dealing with the nozzle flows.

8. Solve the following equations by Gauss-Elimination method:
   2x+2y+4z=18
   x+3y+2z=13
   3x + y + 3z = 14.

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